

We can write  $A \cos \omega t + B \sin \omega t$

as  $C \cos(\omega t - \alpha)$

$$\text{where } C = \sqrt{A^2 + B^2} \quad \tan \alpha = \frac{B}{A}$$

Most important is  $C = \text{amplitude}$

$$C^2 = A^2 + B^2 = \frac{(k - m\omega^2)^2 F_0^2 + (c\omega)^2 F_0^2}{((k - m\omega^2)^2 + (c\omega)^2)^2}$$
$$= \frac{F_0^2}{(k - m\omega^2)^2 + (c\omega)^2}$$

$$C = \text{Amplitude} = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

When the damping  $c$  is not zero, the denominator is always positive.

$$c \neq 0 \Rightarrow (c\omega)^2 > 0 \Rightarrow (k - m\omega^2)^2 + (c\omega)^2 > 0$$

So strictly speaking, there is no "resonance" in the damped system.

But we can still plot  $C$  vs.  $\omega$

$$\text{if } \omega \text{ very small, } C \approx \frac{F_0}{k}$$

$$\text{if } \omega \text{ very large } C \approx 0$$