Forced oscillations II

<u>Damped case</u>: $mx'' + cx' + kx = F_0 \cos \omega t$

Undetermined coefficients suggests to try

$$x(t) = A\cos \omega t + B\sin \omega t$$

$$x'(t) = -\omega A \sin \omega t + \omega B \cos \omega t$$

$$x''(t) = -\omega^2 A \cos \omega t - \omega^2 B \sin \omega t$$

$$mx'' + cx' + kx = \left[(k-m\omega^2)A + c\omega B\right] \cos \omega t + \left[(k-m\omega^2)B - c\omega A\right] \sin \omega t$$

So
$$(k-m\omega^2)A + c\omega B = F_0$$
$$-c\omega A + (k-m\omega^2)B = 0$$

$$B = \frac{c\omega}{k-m\omega^2} A$$

$$(k-m\omega^2)A + \frac{(c\omega)^2}{(k-m\omega^2)} A = F_0$$

$$\left[(k-m\omega^2)^2 + (c\omega)^2\right] A = (k-m\omega^2) F_0$$

$$A = \frac{(k-m\omega^2)F_0}{(k-m\omega^2)^2 + (c\omega)^2} \Rightarrow B = \frac{c\omega F_0}{(k-m\omega^2) + (c\omega)^2}$$