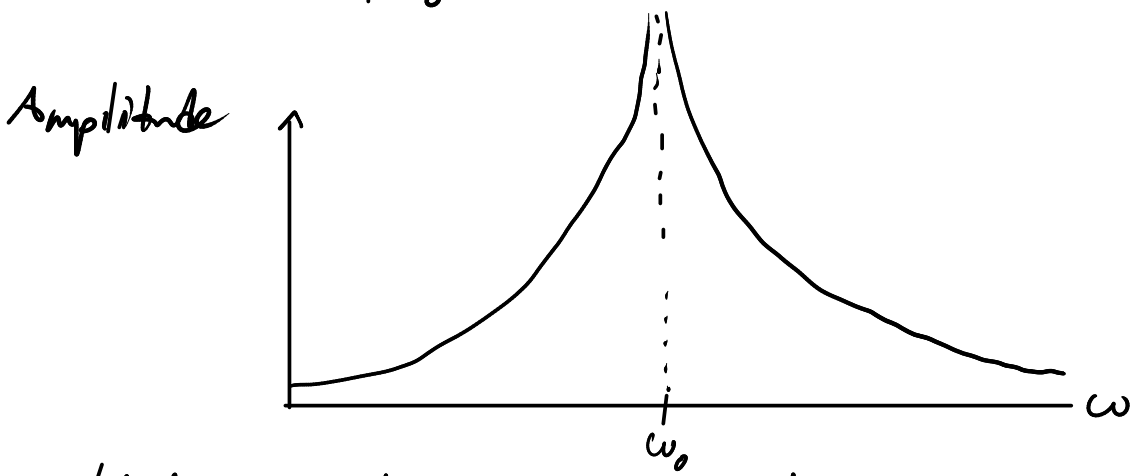
We can draw a picture by plotting the amplitude of the particular solution against the frequency of the driving force

$$x_p(t) = \frac{F_0/m}{\omega_0^2 - \omega^2} \cos \omega t \quad \text{(provided } \omega \neq \omega_0\text{)}$$

$$\text{Amplitude} = \left| \frac{F_0/m}{\omega_0^2 - \omega^2} \right|$$



The plot has a vertical asymptote at $\omega = \omega_0$.
This is the resonance phenomenon.

At $\omega = \omega_0$, the solution is $x_p(t) = \frac{F_0}{2m\omega_0} t \sin \omega_0 t$

It has no finite amplitude because the t factor goes to infinity!

Now, the general solution to the nonhomogeneous equation has other terms.

$$mx'' + kx = 0 \rightsquigarrow x(t) = c_1 \sin \omega_0 t + c_2 \cos \omega_0 t$$