What happens if $\omega = \omega_0$? Driving freq = natural freq.
$k = m\omega^2$

Then we would be dividing by zero! It doesn't make sense! This is (physical) <u>resonance</u>.

And look if $k = m\omega^2$, $(mD^2 + k)(F_0 \cos \omega t)$
$= (-m\omega^2 + k) F_0 \cos \omega t = 0$

So driving force solves the homogeneous equation that is mathematical resonance that we talked about last time.

<u>Resonant case</u>: To solve $mx'' + kx = F_0 \cos(\omega_0 t)$
where $\omega_0 = \sqrt{\frac{k}{m}}$, we try

$$x(t) = At \sin \omega_0 t + Bt \cos \omega_0 t$$
$$x'(t) = A(\omega_0 t \cos \omega_0 t + \sin \omega_0 t) + B(-\omega_0 t \sin \omega_0 t + \cos \omega_0 t)$$

$$x''(t) = A(-\omega_0^2 t \sin \omega_0 t + \omega_0 \cos \omega_0 t + \omega_0 \cos \omega_0 t)$$
$$+ B(-\omega_0^2 t \cos \omega_0 t - \omega_0 \sin \omega_0 t - \omega_0 \sin \omega_0 t)$$

$$m x''(t) + k x(t) = m(x'' + \omega_0^2 x)$$

$$= m(A \cdot 2\omega_0 \cos \omega_0 t - B \cdot 2\omega_0 \sin \omega_0 t)$$

Want $= F_0 \cos \omega_0 t$

so $B = 0$ and $2Am\omega_0 = F_0$

$$x_p(t) = \frac{F_0}{2m\omega_0} t \sin \omega_0 t$$