Undamped case, $c=0$:

$$m x'' + kx = F_0 \cos \omega t$$

Try undetermined coefficients

Nonhomog. term	$F_0 \cos\omega t$	just get $\sin \omega t$ & $\cos\omega t$
1st deriv	$-\omega F_0 \sin \omega t$	
2nd deriv	$-\omega^2 F_0 \cos\omega t$	

So try $x(t) = A \sin\omega t + B\cos\omega t$

$$mx''+kx = -m\omega^2 A\sin\omega t - m\omega^2 B\cos\omega t + kA\sin\omega t + kB\cos\omega t$$
$$= (-m\omega^2+k)A\sin\omega t + (-m\omega^2+k)B\cos\omega t$$

Want this $= F_0\cos\omega t$

Thus, $A=0$ and $(-m\omega^2+k)B = F_0$

so $B = \dfrac{F_0}{k-m\omega^2}$ $\qquad x_p(t) = \dfrac{F_0}{k-m\omega^2}\cos\omega t$

Recall notation $\omega_0 = \sqrt{\frac{k}{m}}$ natural frequency.

$$\frac{F_0 \;\div m}{k-m\omega^2 \,/m} = \frac{F/m}{\frac{k}{m}-\omega^2} = \frac{F/m}{\omega_0^2-\omega^2}$$

So the amplitude of the particular solution depends on the difference between the driving frequency and the natural frequency.