

Then find $x(t)$ from $\frac{dx}{dt} = v(t)$, which we now know

$$x(t) = \int v(t) dt + \textcircled{D}$$

another constant of integration

All told, our solution has two constants of integration
We will need two initial conditions as well

Eg constant acceleration $a(t) = -g$ (downward gravity)

$$v(t) = \int a(t) dt = \int -g dt = -gt + C$$

$$x(t) = \int v(t) dt = \int (-gt + C) dt = -\frac{1}{2}gt^2 + Ct + D$$

↑ ↑
two constants

General solution $x(t) = -\frac{1}{2}gt^2 + Ct + D$

Now $x(0) = D$ so D represents initial position

And $v(0) = C$ so C represents initial velocity

If we specify the initial position and velocity, we will get a particular solution.