How about $(D^2+D+2)\,y = x^2e^x$

Nonhomogeneous term $= x^2e^x$

First deriv $= x^2e^x + 2xe^x$

Second deriv $= x^2e^x + 2xe^x + 2xe^x + 2e^x$

Third deriv = gonna get stuff like x^2e^x, xe^x and e^x.

So try $y_p = Ax^2e^x + Bxe^x + Ce^x$

Want $(D^2+D+2)(Ax^2e^x + Bxe^x + Ce^x) = x^2e^x$

$= A(x^2e^x + 4xe^x + 2e^x) + B(xe^x + 2e^x) + Ce^x$

$+ A(x^2e^x + 2xe^x) + B(xe^x + e^x) + Ce^x$

$+ 2Ax^2e^x + 2Bxe^x + 2Ce^x$

$= 4Ax^2e^x + (4A + B + 2A + B + 2B)xe^x + (2A + 2B + C + B + C + 2C)e^x$

$4A = 1$ $\quad A = 1/4$

$6A + 4B = 0$ $\quad 3A + 2B = 0$ $\quad B = -\frac{3}{8}$

$2A + 3B + 4C = 0$ $\quad \frac{1}{2} - \frac{9}{8} + 4C = 0$ $\quad \frac{4}{8} - \frac{9}{8} + 4C = 0$

$-\frac{5}{8} + 4C = 0$

$C = \frac{5}{32}$

$$\boxed{y_p = \frac{1}{4}x^2e^x - \frac{3}{8}xe^x + \frac{5}{32}e^x}$$