Thus $y_p = \frac{1}{2}\sin(x) - \frac{1}{2}\cos(x)$ is a particular solution.

The general solution is gotten by adding to this the general solution of the homogeneous equation $(D^2+D+2)y=0$

$$r^2+r+2=0 \qquad r = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1}{2} \pm i\frac{\sqrt{7}}{2}$$

solutions $y_c = e^{-\frac{1}{2}x}\left(c_1 \cos\left(\frac{\sqrt{7}}{2}x\right) + c_2 \sin\left(\frac{\sqrt{7}}{2}x\right)\right)$

General solution of $(D^2+D+2)y = \sin(x)$ is

$$y = \underbrace{e^{-\frac{1}{2}x}\left(c_1 \cos\left(\tfrac{\sqrt{7}}{2}x\right) + c_2 \sin\left(\tfrac{\sqrt{7}}{2}x\right)\right)}_{y_c} + \underbrace{\tfrac{1}{2}\sin(x) - \tfrac{1}{2}\cos(x)}_{y_p}$$

$$p(D)(y_c+y_p) = p(D)y_c + p(D)y_p = 0 + \sin(x) \quad \checkmark$$

We can make this "trial and error" method more systematic, but the question is
"What am I supposed to try?"

Somewhat imprecise but useful suggestion:
"Try a linear combination of all the terms that appear in the derivatives of the nonhomogeneous term."

Let's consider $p(D) = D^2+D+2$ again, but now we want to solve
$$p(D)y = x^3$$