

Critical damping: $c^2 = 4km$

$$m\ddot{x} + c\dot{x} + kx = 0$$

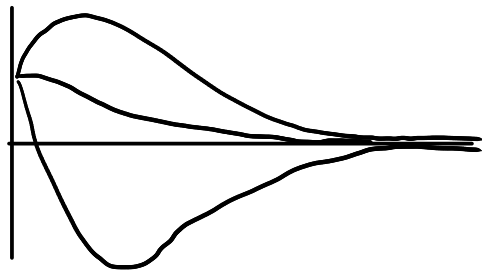
$$r = \frac{-c \pm \sqrt{c^2 - 4km}}{2m} = \frac{-c}{2m}$$

$r = -\gamma$ repeated root!

So basic solutions are $e^{-\gamma t}$, $te^{-\gamma t}$

General solution $x(t) = e^{-\gamma t} (c_1 + c_2 t)$

Plot



No actual oscillation!

Overdamped

$$c^2 > 4km$$

$$r = \frac{-c \pm \sqrt{c^2 - 4km}}{2m}$$

$$r = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

Basic solutions $e^{(-\gamma + \sqrt{\gamma^2 - \omega_0^2})t}$, $e^{(-\gamma - \sqrt{\gamma^2 - \omega_0^2})t}$

General solution $x(t) = e^{-\gamma t} (c_1 e^{\sqrt{\gamma^2 - \omega_0^2} t} + c_2 e^{-\sqrt{\gamma^2 - \omega_0^2} t})$

Could also be written $x(t) = e^{-\gamma t} (A \cosh(\sqrt{\gamma^2 - \omega_0^2} t) + B \sinh(\sqrt{\gamma^2 - \omega_0^2} t))$

Plot

