

General

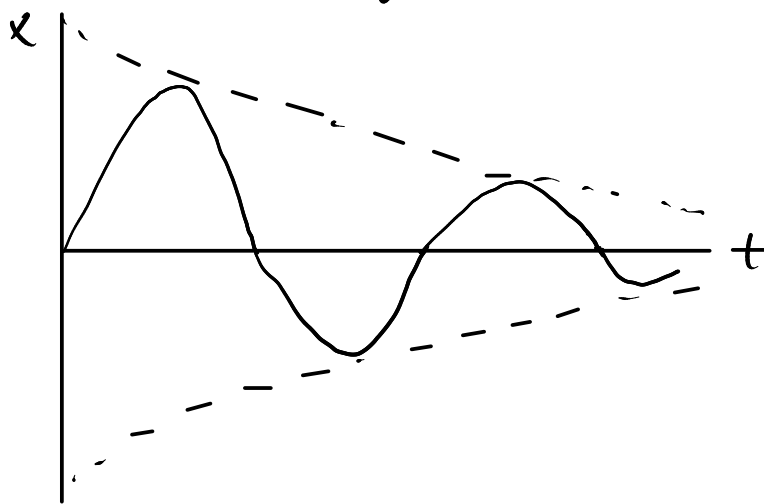
$$\text{Solution } x(t) = A e^{-\gamma t} \cos(\sqrt{\omega_0^2 - \gamma^2} t) + B e^{-\gamma t} \sin(\sqrt{\omega_0^2 - \gamma^2} t)$$

$$\text{or } x(t) = C e^{-\gamma t} \cos(\sqrt{\omega_0^2 - \gamma^2} t - \alpha)$$

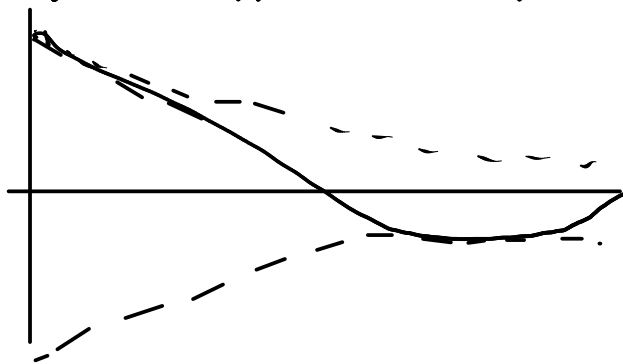
Let $\omega = \sqrt{\omega_0^2 - \gamma^2}$ This is the new "frequency" less than the natural frequency.

The "Amplitude" $C e^{-\gamma t}$ decays in time

Plot



As $\gamma = \frac{c}{2m}$ get bigger, the frequency get lower



There is a critical damping coefficient where

$$\omega = \sqrt{\omega_0^2 - \gamma^2} = 0 \quad \omega_0 = \gamma \quad \sqrt{\frac{k}{m}} = \frac{c}{2m}$$

$$\text{That is } c^2 = 4km$$