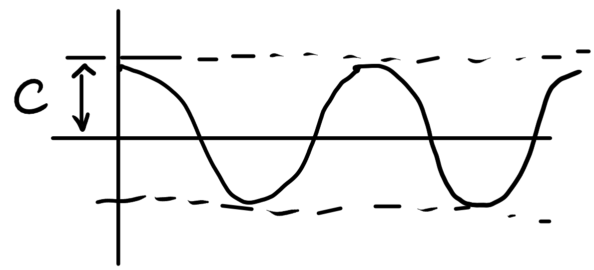
Thus the general solution can also be written

$$x(t) = C\cos(\omega_0 t - \alpha) \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

C = amplitude
α = phase shift

Plot:



What if damping $c \neq 0$?

$$m x'' + c x' + kx = 0 \qquad r = \frac{-c \pm \sqrt{c^2 - 4km}}{2m}$$
$$m r^2 + cr + k = 0$$

It makes a difference if
$c^2 - 4km < 0$ underdamped
$c^2 - 4km = 0$ critically damped
$c^2 - 4km > 0$ overdamped.

Underdamped $c^2 - 4km < 0$ still have complex roots

$$r = \frac{-c}{2m} \pm i\,\frac{\sqrt{4km - c^2}}{2m}$$

Abbreviations: $\gamma = \frac{c}{2m} \qquad \omega_0 = \sqrt{\frac{k}{m}}$

Thus $\frac{\sqrt{4km - c^2}}{2m} = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}} = \sqrt{\omega_0^2 - \gamma^2}$

$$r = -\gamma \pm i\sqrt{\omega_0^2 - \gamma^2}$$