Simplest case: No damping $c=0$

$$mx'' + kx = 0$$

Characteristic equation $mr^2 + k = 0$ $\qquad r^2 = -\frac{k}{m}$ $\qquad r = \pm i\sqrt{\frac{k}{m}}$

Complex roots: A complex solution is $x(t) = e^{i\sqrt{\frac{k}{m}}t}$

Real part: $\cos\left(\sqrt{\frac{k}{m}}t\right)$ $\qquad$ Imag. part $\sin\left(\sqrt{\frac{k}{m}}t\right)$

General solution $x(t) = c_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + c_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$

$\sqrt{\frac{k}{m}}$ has units of $(\text{time})^{-1}$ it is called the natural frequency

$$\boxed{\text{Natural frequency: } \omega_0 = \sqrt{\frac{k}{m}}}$$

This is an angular frequency, so the period is $T = \frac{2\pi}{\omega_0}$.

Note that an expression like
$$A\cos(\theta) + B\sin(\theta)$$
is equivalent to one like
$$C\cos(\theta - \alpha) = C(\cos\alpha \cos\theta + \sin\alpha \sin\theta)$$

if we have the correspondence of vectors
$$\langle A, B\rangle = \langle C\cos\alpha, C\sin\alpha\rangle$$
that is if (C, α) is the polar representation of $\langle A, B\rangle$

$$C = \sqrt{A^2 + B^2} \qquad \tan\alpha = \frac{B}{A}$$