Then we can consider the differential equation

$$(aD^2+bD+c)\,y=0$$

$$\big(D-(p+qi)\big)\big(D-(p-qi)\big)\,y=0$$

$$\boxed{\begin{aligned} p&=\frac{-b}{2a}\\ q&=\frac{\sqrt{4ac-b^2}}{2a}\end{aligned}}$$

Solution of $\big(D-(p+qi)\big)\,y=0$ is $y=e^{(p+qi)x}$

What is that? $e^{(p+qi)x}=e^{px+iqx}=e^{px}e^{iqx}$

$$=e^{px}\big(\cos(qx)+i\sin(bx)\big)$$

Since this ↗ solves $(aD^2+bD+c)y=0$, so do its real and imaginary parts

$$\left.\begin{aligned} \mathrm{Re}\left(e^{(p+qi)x}\right)&=e^{px}\cos(qx)\\ \mathrm{Im}\left(e^{(p+qi)}x\right)&=e^{px}\sin(qx)\end{aligned}\right\}\ \text{these both solve } (aD^2+bD+c)\,y=0$$

The <u>general solution</u> is $\boxed{y(x)=c_1e^{px}\cos(qx)+c_2e^{px}\sin(qx)}$

<u>Example</u> find general solution of $(D^2-D+2)\,y=0$

Characteristic equation

$r^2-r+2=0$ $\qquad r=\dfrac{1\pm\sqrt{1-4\cdot 2}}{2}=\dfrac{1}{2}\pm i\dfrac{\sqrt{7}}{2}$

A Complex solution is $y=e^{\left(\frac{1}{2}+i\frac{\sqrt{7}}{2}\right)x}=e^{\frac{1}{2}x}e^{i\frac{\sqrt{7}}{2}x}=$

$$=e^{\frac{1}{2}x}\left(\cos\left(\frac{\sqrt{7}}{2}x\right)+i\sin\left(\frac{\sqrt{7}}{2}x\right)\right)$$

Real and Imaginary parts $y_1=e^{\frac{1}{2}x}\cos\left(\frac{\sqrt{7}}{2}x\right)$ $\quad y_2=e^{\frac{1}{2}x}\sin\left(\frac{\sqrt{7}}{2}x\right)$

<u>Real</u> General solution: $\boxed{y(x)=c_1e^{\frac{1}{2}x}\cos\left(\frac{\sqrt{7}}{2}x\right)+c_2e^{\frac{1}{2}x}\sin\left(\frac{\sqrt{7}}{2}x\right)}$