But these are complex valued solutions, we want real-valued ones

General principle: If a <u>complex-valued</u> function solves a differential equation, and the equation has <u>real</u> coefficients, then the <u>real</u> and <u>imaginary</u> parts both solve the same differential equation.

So because $e^{ix} = \cos(x) + i\sin(x)$ solves $\underbrace{(D^2+1)}_{\text{real}}y = 0$

We know that

$$\left.\begin{array}{lr}\text{Real part} & Re(e^{ix}) = \cos(x) \\ \text{and Imag. part} & Im(e^{ix}) = \sin(x)\end{array}\right\} \text{ also solve } (D^2+1)y=0$$

Indeed
$$(D^2+1)[\cos(x)] = -\cos(x) + \cos(x) = 0$$
$$(D^2+1)[\sin(x)] = -\sin(x) + \sin(x) = 0$$

Also e^{-ix} solves $(D^2+1)y=0$ so

$$\left.\begin{array}{lr} & Re(e^{-ix}) = \cos(x) \\ \text{and} & Im(e^{-ix}) = -\sin(x)\end{array}\right\} \text{ also solve, but these are not really new.}$$

General irreducible quadratic has a pair of complex conjugate roots.

Consider $ar^2+br+c=0$ Assume $b^2-4ac<0$

$$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm i\,\frac{\sqrt{4ac-b^2}}{2a} = p \pm qi$$

where $p = \frac{-b}{2a}$ and $q = \frac{\sqrt{4ac-b^2}}{2a}$