Look at $(D-i)y=0 \iff y'-iy=0 \iff y'=iy$

Since this equation has a complex coefficient (i),
The solution cannot be an "ordinary" real-valued function
It must be a complex-valued function.

Complex-valued function. $\begin{cases} x & \text{independent variable is real} \\ y=f(x) & \text{dependent variable is allowed to be complex} \end{cases}$

The solution is $y=e^{ix}$
Does this make sense? Well by chain rule

$$y' = \frac{d}{dx}\left[e^{ix}\right] = e^{ix}\frac{d}{dx}\left[ix\right] = e^{ix}\, i = i e^{ix} = iy$$

In fact not only does e^{ix} exist, we have a formula for it!

EULER'S FORMULA: $e^{ix} = \cos(x) + i\sin(x)$

Feynman: "This is our jewel." Ch. 22 of lectures on physics

Check $\frac{d}{dx}\left[\cos(x)+i\sin(x)\right] = -\sin(x)+i\cos(x)$

$i\left[\cos(x)+i\sin(x)\right] = i\cos(x)+i^2\sin(x)$

these are equal b/c $i^2=-1$!

Now look at $(D+i)y=0 \rightarrow y'=-iy \longrightarrow y=e^{-ix}$

$$e^{-ix} = e^{i(-x)} = \cos(-x)+i\sin(-x) = \cos(x) - i\sin(x)$$

Thus both e^{ix} and e^{-ix} solve $(D^2+1)y=0$