Complex Roots of the Characteristic equation

Fundamental example: what is the general solution of the

Differential equation $\qquad y'' + y = 0$

that is, $\qquad (D^2+1)y = 0 \qquad D = \frac{d}{dx}$

Characteristic equation $\qquad p(r) = r^2 + 1 = 0$

$$r^2 = -1$$

$$r = \pm\sqrt{-1} = \pm i$$

$i = \sqrt{-1}$ is called the imaginary unit

E.g. $\quad 5$: real number $\qquad 2i$: imaginary number

$5+2i$: complex number.

A complex number $a+bi$ has real part a and imaginary part b

Real part $\qquad Re(5+2i) = 5$

Imaginary part $\qquad Im(5+2i) = 2$

Thus r^2+1 factors

$$p(r) = r^2 + 1 = (r-i)(r+i)$$

And the differential operator factors

$$p(D) = D^2 + 1 = (D-i)(D+i)$$

So in order to solve $(D^2+1)y = 0$

we can first solve $\quad (D-i)y = 0$

$$(D+i)y = 0$$