

What if only some roots are repeated?

$$\text{Eg. } (D-5)(D-2)^2 = (D-5)(D^2-4D+4) = D^3-9D^2+24D-20$$

$$\text{Solutions of } (D-5)y=0 \text{ is } y_1 = e^{5x}$$

$$\text{Solutions of } (D-2)^2 y=0 \text{ is } y_2 = e^{2x} \quad y_3 = x e^{2x}$$

By the lemma, all 3 of these functions solve

$$Ly = (D^3-9D^2+24D-20)y = 0$$

They are linearly independent, so the general solution is

$$y(x) = c_1 e^{5x} + c_2 e^{2x} + c_3 x e^{2x}$$