

Idea: try  $f(x)e^{ax}$ , and see if it simplifies.

$$\begin{aligned} (D-a)[f(x)e^{ax}] &= D[f(x)e^{ax}] - af(x)e^{ax} \\ &= f'(x)e^{ax} + f(x)ae^{ax} - af(x)e^{ax} = f'(x)e^{ax} \\ &= (D[f(x)])e^{ax} \end{aligned}$$

Fact  $(D-a)[fe^{ax}] = (Df)e^{ax}$

Then  $(D-a)^2[fe^{ax}] = (D^2f)e^{ax}$

$(D-a)^n[fe^{ax}] = (D^n f)e^{ax}$

Thus  $0 = (D-a)^n[fe^{ax}]$  as long as  $D^n f = 0$ !

What are the solutions of  $D^n f = 0$ ? Polynomials of degree  $n-1$ !

$$D^n (c_0 + c_1x + c_2x^2 + \dots + c_{n-1}x^{n-1}) = 0!$$

But  $D^n[x^n] = n! \neq 0$

Conclusion: Some solutions of  $(D-a)^n y = 0$  are  
 $y_1 = e^{ax}$ ,  $y_2 = xe^{ax}$ ,  $y_3 = x^2e^{ax}$ , ...,  $y_n = x^{n-1}e^{ax}$

The general solution is  $y(x) = (c_0 + c_1x + c_2x^2 + \dots + c_{n-1}x^{n-1})e^{ax}$