Key fact: For constant <u>coefficient</u> differential operators, the order of the factors does not matter.

$$(D-a)(D-b) = D^2-(a+b)D+ab = (D-b)(D-a)$$

[This isn't true for nonconstant coefficients like $(D-x)$]

<u>Lemma</u> if $L=f(D)g(D)$ for polynomials $f(r)$ and $g(r)$

<u>Then</u> any function y_1 satisfying $f(D)\cdot y=0$ also satisfies $Ly=0$
<u>and</u> any function y_2 satisfying $g(D)\cdot y=0$ also satisfies $Ly=0$

<u>Proof</u> $Ly_1 = f(D)g(D)y_1 = g(D)f(D)y_1$ b/c order doesn't matter here
$= g(D)\left[f(D)y_1\right] = g(D)[0] = 0$

And $Ly_2 = f(D)g(D)y_2 = f(D)\left[g(D)\cdot y_2\right] = f(D)[0] = 0$.

This is useful b/c it says we can solve the two equations $f(D)y=0$ and $g(D)y=0$ and get solutions to $Ly=0$.

- Case of repeated roots. The "worst case" would be just one root of the characteristic equation

$$p(r)=(r-a)^n=0 \quad \text{ie} \quad L=p(D)=(D-a)^n$$

Q: What are solutions of $(D-a)^n y = 0$?

We know $y=e^{ax}$ works, since $(D-a)e^{ax}=0$