A general constant coefficient linear differential operator looks like

$$L = a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0$$

$$L \cdot y = (a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0) \cdot y$$

$$= a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y$$

So this is a new notation for the LHS's of the differential equations we want to consider.

Observe the similarity between

Differential operator $\quad L = a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0$

Characteristic polynomial $\quad p(r) = a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0$

So we can write $L = p(D)$ if we think of D as substituted for r.

Just as we can factor polynomials, we can factor differential operators.

Eg. Factor $\quad D^2 - 2D + 1 = (D-1)^2$

$D^2 + 5D + 1$

$r^2 + 5r + 1 \qquad \dfrac{-5 \pm \sqrt{25-4}}{2} = \dfrac{-5 \pm \sqrt{21}}{2}$

$$D^2 + 5D + 1 = \left[D - \left(\frac{-5+\sqrt{21}}{2}\right)\right]\left[D - \left(\frac{-5-\sqrt{21}}{2}\right)\right]$$