Example $y^{(3)} - 3y'' + 2y' = 0$

Characteristic equation $r^3 - 3r^2 + 2r = 0$

$$r(r^2 - 3r + 2) = 0$$
$$r(r-1)(r-2) = 0$$

So solutions $r_1 = 0 \quad r_2 = 1 \quad r_3 = 2$ Real, distinct

general solution $y(x) = c_1 e^{0x} + c_2 e^{x} + c_3 e^{2x} = c_1 + c_2 e^{x} + c_3 e^{2x}$

What if a root is repeated?

Eg $y'' - 2y' + y = 0$
$r^2 - 2r + 1 = 0$
$(r-1)^2 = 0 \quad \leftarrow$ double solution $r_1 = 1 \quad r_2 = 1$

So $y = e^{r_1 x} = e^{x}$ is a solution

The general solution is NOT $y(x) = c_1 e^{x} + c_2 e^{x}$

because e^{x} and e^{x} are not linearly independent!

We need ANOTHER solution that CANNOT be found using the characteristic equation.

This requires some work, so we are going to introduce some theory that will make it more comprehensible.