Constant coefficient linear homogeneous equations

This is an equation like

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are <u>constants</u>.

The basic idea is to try a solution of the form e^{rx}

$$y = e^{rx} \quad y' = re^{rx} \quad y'' = r^2 e^{rx}, \ldots, \quad y^{(n)} = r^n e^{rx}$$

Then $a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$
becomes

$$a_n r^n e^{rx} + a_{n-1} r^{n-1} e^{rx} + \cdots + a_1 r e^{rx} + a_0 e^{rx} = 0$$

$$\left(a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0\right) e^{rx} = 0$$

Thus e^{rx} will be a solution if

$$\boxed{a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0 = 0}$$ Characteristic Equation.

To get Characteristic equation from the original homogeneous linear differential equation, replace
$y \to 1$, $y' \to r$, $y'' \to r^2$, etc $y^{(n)} \to r^n$

To summarize: If r solves characteristic equation, then $y = e^{rx}$ solves the differential equation.