Proof $(y_c+y_p)'' + p(x)(y_c+y_p)' + q(x)(y_c+y_p)$

$$= y_c'' + y_p'' + p(x)\,y_c' + p(x)y_p' + q(x)\,y_c + q(x)\,y_p$$

$$= \left[y_c'' + p(x)y_c' + q(x)y_c\right] + \left[y_p'' + p(x)y_p' + q(x)\,y_p\right]$$

$$= \quad 0 \quad + \quad f(x)$$

$$= f(x)$$

QED.

General solutions Consider nth order linear nonhomogeneous DE

$$y^{(n)} + p_1(x)\,y^{(n-1)} + \cdots + p_n(x)y = f(x)$$

If y_p is any particular solution of this equation,

and $y_1, y_2, \ldots, y_n$ are n linearly independent solutions

of the homogeneous equation

$$y^{(n)} + p_1(x)\,y^{(n-1)} + \cdots + p_n(x)y = 0$$

$$y(x) = y_p(x) + c_1 y_1(x) + c_2 y_2(x) + \cdots + c_n y_n(x)$$

is the general solution of the nonhomogeneous equation.