

The principle of superposition does not hold for nonhomogeneous equations.

so for example $2 \cdot \left(-\frac{1}{5} \sin(x)\right)$ is not a solution

But a related principle does apply

If we add a solution of the homogeneous equation to a solution of the nonhomogeneous equation, we get a solution of the nonhomogeneous equation

let y_p be a solution of the nonhomogeneous equation

$$y'' + p(x)y' + q(x)y = f(x) \quad \text{"particular solution"}$$

so let y_c be a solution of the homogeneous equation

$$y'' + p(x)y' + q(x)y = 0$$

Then $y_c + y_p$ satisfies $y'' + p(x)y' + q(x)y = f(x)$

* This also works for higher order equations.

- Possible analogy:
Even and odd numbers

homog. solutns nonhomog. solutns

$$\text{Even} + \text{Even} = \text{Even}$$

$$\text{Odd} + \text{Even} = \text{Odd}$$

$$\text{Odd} + \text{Odd} \neq \text{Odd}$$

Odd + Odd = even

This is where the analogy breaks down
So don't take it too seriously