Nonhomogeneous linear equations, constant coefficient equations

[First we finish showing example of linear independence]

Nonhomogeneous linear diff. eqns.

$$y^{(n)} + p_1(x) y^{(n-1)} + \cdots + p_{n-1}(x) y' + p_n(x) y = f(x)$$

Eg $\quad y'' - 4y = \sin(x)$

We cannot solve using characteristic equation.

Try: $y = e^{rx}$ $\qquad r^2 e^{rx} - 4e^{rx} = \sin(x)$? can't work

Later, in section 3.5, we will learn a technique to solve this equation. It turns out that the solution is

$$y_p(x) = -\frac{1}{5}\sin(x)$$

Check: $-\frac{1}{5}(-\sin(x)) - 4\left(-\frac{1}{5}\right)\sin(x) = \dfrac{\sin(x) + 4\sin(x)}{5} = \sin(x)$

Recall: The solutions of $y'' - 4y = 0$

are $\quad y_1 = e^{2x}$, $y_2 = e^{-2x}$, with general solution

$$y = c_1 e^{2x} + c_2 e^{-2x}$$

So the solution of the nonhomogeneous equation looks completely different from that of the homogeneous one.