The n solutions need to be <u>linearly independent</u>

It easier to define when a collection of n functions is <u>linearly dependent</u> (a/k/a "<u>redundant</u>")

- A collection of n functions $y_1, \ldots, y_n$ is <u>linearly dependent</u> (or <u>redundant</u>) if there exist constants $c_1, \ldots, c_n$, NOT ALL $= 0$, such that
$$c_1 y_1 + c_2 y_2 + \cdots + c_3 y_3 = 0 \quad \text{for } \underline{\text{all } x}.$$

- n functions $y_1 \ldots y_n$ are <u>linearly independent</u> if they are not <u>linearly dependent</u>

Eg. e^x, e^{-x} and e^{3x} are linearly independent.

Suppose
$$c_1 e^x + c_2 e^{-x} + c_3 e^{3x} = 0$$
Plug in $x = 0$
$$c_1 + c_2 + c_3 = 0$$
Take derivative and plug in $x = 0$
$$c_1 e^x - c_2 e^{-x} + 3c_3 e^{3x} = 0$$
$$c_1 - c_2 + 3c_3 = 0$$
Take 2nd deriv and plug in 0
$$c_1 + c_2 + 9c_3 = 0$$

$$8c_3 = 0 \Rightarrow c_3 = 0$$
$$c_1 = c_2$$
$$c_1 + c_2 = 0 \rightarrow c_1 = 0, \quad c_2 = 0.$$

Since all coefficients are forced to be zero, the functions are linearly independent.

Not independent: $e^x, e^{-x}, \sinh(x) = \dfrac{e^x - e^{-x}}{2}$