In this case the characteristic equation factors nicely.

$$r^3 - 3r^2 - r + 3 = (r^2 - 1)(r-3) = (r-1)(r+1)(r-3)$$

so the roots are $r = 1, -1, 3$

Thus $y_1 = e^x$ $y_2 = e^{-x}$ and $y_3 = e^{3x}$ are solutions of the Diff Eqn. $y''' - 3y'' - y' + 3y = 0$

Linear homogeneous Diff eqn of any order satisfy <u>principle of superposition</u>

So a general solution is $y = c_1 e^x + c_2 e^{-x} + c_3 e^{3x}$ where c_1, c_2, and c_3 are constants.

Why is this the <u>complete</u> general solution?

For a third-order equation, we need to specify 3 initial conditions

$$\begin{cases} y(a) = b_0 \\ y'(a) = b_1 \\ y''(a) = b_2 \end{cases}$$

For n-th order, we need to specify n initial conditions

$$\left.\begin{aligned} y(a) &= b_0 \\ y'(a) &= b_1 \\ y''(a) &= b_2 \\ &\vdots \\ y^{(n-1)}(a) &= b_{n-1} \end{aligned}\right\} \; n \text{ conditions.}$$

So we need n constants

→ we need n distinct solutions