Eg. for $y'' - 4y = 0$

$y = c_1 e^{2x} + c_2 e^{-2x}$ is a complete general solution

So is $y = d_1 \sinh(2x) + d_2 \cosh(2x)$

or even $y = k_1 e^{2x} + k_2 \sinh(2x)$

Higher order linear differential equations

Notation: $y^{(n)} = \underbrace{y''^{\cdots\prime}}_{n \text{ primes}} = \dfrac{d^n y}{dx^n} =$ nth derivative of y wrt x.

A typical n-th order linear DE looks like

$$y^{(n)} + p_1(x) y^{(n-1)} + p_2(x) y^{(n-2)} + \cdots + p_{n-1}(x) y' + p_n(x) y = q(x)$$

The equation is <u>homogeneous</u> if $q(x) = 0$.

Let's do a third-order example:

$$y''' - 3y'' - y' + 3y = 0$$

Behind the scenes

$(r-1)(r+1)(r-3)$

$(r^2-1)(r-3)$

$r^3 - 3r^2 - r + 3$

For constant coefficient linear homogeneous, we can try $y = e^{rx}$ again.

$$y' = re^{rx} \quad y'' = r^2 e^{rx} \quad y''' = r^3 e^{rx}$$

$$r^3 e^{rx} - 3r^2 e^{rx} - re^{rx} + 3e^{rx} = 0$$

$$\boxed{r^3 - 3r^2 - r + 3 = 0}$$ CHARACTERISTIC EQN.