This is a 2×2 linear system for c_1 and c_2

The coefficient matrix is $\begin{bmatrix} y_1(a) & y_2(a) \\ y_1'(a) & y_2'(a) \end{bmatrix}$

The system is always solvable if it is nonsingular, which means the determinant.

$$\begin{vmatrix} y_1(a) & y_2(a) \\ y_1'(a) & y_2'(a) \end{vmatrix} = y_1(a)y_2'(a) - y_2(a)y_1'(a) \text{ \underline{is not zero}.}$$

This determinant is called the Wronskian of y_1 and y_2 at a.

It turns out that this condition is equivalent to saying y_1 and y_2 are <u>linearly independent</u>.

<u>Def</u> Two functions y_1 and y_2 are <u>linearly independent</u> if they are not proportional:

$$y_1 \neq C y_2$$
$$y_2 \neq C y_1$$

Otherwise they are <u>linearly dependent</u>.

<u>Examples</u> $\sin(x), \cos(x)$; e^x, e^{-2x} ; e^x, xe^x, ...

<u>Non-examples</u>: $e^x, 2e^x$; 0, $\sin(x)$, ...