

The question is, how do we know when we've found "enough" solutions?

We start with a theoretical fact

Theorem (Existence and uniqueness for 2nd order linear equations)

Suppose  $p(x)$ ,  $q(x)$ , and  $f(x)$  are continuous on an interval  $I$  and let  $a$  be a point of  $I$ .

THEN the initial value problem

$$\left\{ \begin{array}{l} y'' + p(x)y' + q(x)y = f(x) \\ y(a) = b_0 \\ y'(a) = b_1 \end{array} \right\}$$

has a unique solution defined on  $I$ .

Rephrasing, we should get a unique solution for any pair of initial value and initial derivative.

We have found "enough" solutions when we have enough constants to solve any such initial value problem.

Suppose we find two solutions  $y_1(x)$  and  $y_2(x)$  to the homogeneous equation  $y'' + p(x)y' + q(x)y = 0$ .

Need to be able to solve for  $y(a) = b_0$   
 $y'(a) = b_1$

$$\text{or } \left\{ \begin{array}{l} c_1 y_1(a) + c_2 y_2(a) = b_0 \\ c_1 y_1'(a) + c_2 y_2'(a) = b_1 \end{array} \right\}$$