This is the <u>characteristic equation</u>. It is the condition that r must satisfy in order for $y = e^{rx}$ to solve the original DE.

<u>Example</u>: $$y'' - 4y' + 3y = 0$$

Try e^{rx}
$$r^2 e^{rx} - 4re^{rx} + 3e^{rx} = 0$$
$$r^2 - 4r + 3 = 0$$
$$(r-3)(r-1) = 0$$
so $r = 3$ or 1.

Thus $y_1 = e^{3x}$ and $y_2 = e^{1x} = e^x$ are solutions!

Now by the principle of superposition for linear homogeneous equations, we know that
$$y(x) = c_1 y_1 + c_2 y_2 = c_1 e^{3x} + c_2 e^x$$
is also a solution, for any constants c_1 and c_2.

In general, if r_1 and r_2 are solutions of the characteristic equation $ar^2 + br + c = 0$, And the solutions are real and distinct, then
$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$
is a solution of
$$ay'' + by' + cy = 0$$