Second order linear equations continued.

Without further delay, let's see how to solve a 2nd order linear homogeneous equation.

We can do this when the equation has constant coefficients

Consider $ay'' + by' + cy = 0$

What are some solutions?

Recall 1st order case $ay' + by = 0$

$$\frac{y'}{y} = -\frac{b}{a} \longrightarrow \int \frac{dy}{y} = \int \left(-\frac{b}{a}\right) dx$$

$$\ln|y| = \left(-\frac{b}{a}\right)x + C \longrightarrow y = De^{\left(-\frac{b}{a}\right)x}$$

So, let's try $y(x) = e^{rx}$, where r is a constant to be determined.

$$y = e^{rx}, \quad y' = re^{rx}, \quad y'' = r^2 e^{rx}$$

So $ay'' + by' + cy = 0$ becomes

$$ar^2 e^{rx} + bre^{rx} + ce^{rx} = 0$$

Since e^{rx} is never zero, we can divide by it:

$$ar^2 + br + c = 0.$$