

Principle of Superposition  
for second order linear homogeneous equations.

Consider  $A(x)y'' + B(x)y' + C(x)y = 0$   
Suppose two functions  $y_1(x)$  and  $y_2(x)$  satisfy this equation

THEN so does  $c_1y_1 + c_2y_2$ , where  $c_1$  and  $c_2$  are any constants.

Proof: We know  $Ay_1'' + By_1' + Cy_1 = 0$   
and  $Ay_2'' + By_2' + Cy_2 = 0$

So look at

$$\begin{aligned} & A(c_1y_1 + c_2y_2)'' + B(c_1y_1 + c_2y_2)' + C(c_1y_1 + c_2y_2) \\ &= Ac_1y_1'' + Ac_2y_2'' + Bc_1y_1' + Bc_2y_2' + Cc_1y_1 + Cc_2y_2 \\ &= c_1(Ay_1'' + By_1' + Cy_1) + c_2(Ay_2'' + By_2' + Cy_2) \\ &= c_1 \cdot 0 + c_2 \cdot 0 = 0 \quad \text{QED} \end{aligned}$$

Example: since  $e^{2x}$  and  $e^{-2x}$  solve  $y'' - 4y = 0$

so do:

- $e^{2x} + e^{-2x}$

- $e^{2x} - e^{-2x}$

- $15e^{2x}$

- $\pi e^{2x} + 17e^{-2x}$  etc.