Physically, we know that we need to specify the initial position and the initial velocity.

So the general solution of a second order equation should depend on two undetermined constants, which may later be fixed by initial conditions.

FACT (to be understood)
The general solution of

$$y'' - 4y = 0$$

is

$$y(x) = c_1 e^{2x} + c_2 e^{-2x}$$

Problem: use the "FACT" to solve the initial value problem

$$\begin{cases} y'' - 4y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Solution: Need to find c_1 and c_2

$$y(x) = c_1 e^{2x} + c_2 e^{-2x}$$
$$y'(x) = 2c_1 e^{2x} - 2c_2 e^{-2x}$$

$$1 = y(0) = c_1 e^0 + c_2 e^0 = c_1 + c_2$$

$$0 = 2c_1 e^0 - 2c_2 e^0 = 2c_1 - 2c_2$$

Solve for c_1 and c_2 : get $c_1 = c_2 = \frac{1}{2}$

So $y(x) = \frac{1}{2} e^{2x} + \frac{1}{2} e^{-2x}$ is the particular solution to the IVP.