

Let's talk about solutions:

Example  $y'' - 4y = 0$

Here are some solutions

$$y_1(x) = e^{2x} : \text{ indeed } (e^{2x})'' = 2(e^{2x})' = 4e^{2x}$$
$$\text{so } (e^{2x})'' - 4(e^{2x}) = 0$$

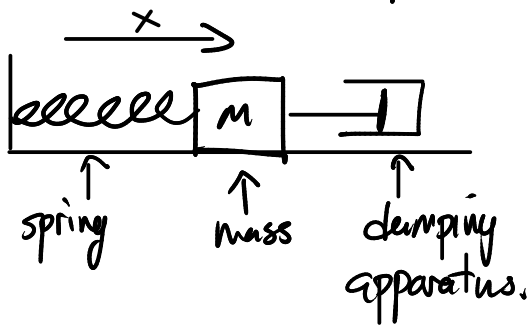
Also  $y_2(x) = e^{-2x} : \text{ indeed } (e^{-2x})'' = -2(e^{-2x})' = (-2)^2 e^{-2x}$

$$\text{so } (e^{-2x})'' - 4(e^{-2x}) = 0.$$

There are even other solutions,  
such as  $\sinh(2x)$ ,  $\cosh(2x)$ , and more.

We want to understand the "space" or "set" of all possible solutions, this is our goal.

Physical example: Damped oscillation



$$\text{Spring force} = -kx$$

$$\text{damping force} = -cv$$

Newton's 2<sup>nd</sup>:

$$ma = -kx - cv$$

$$ma + cv + kx = 0$$

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = 0$$

This is second order linear (and homogeneous)!