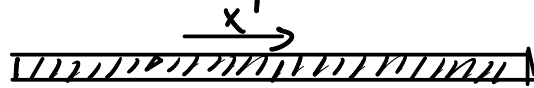


What about the equation $(y')^2 + (y)^2 = -1$?
 No (real) solution!

So existence of solutions is sometimes in doubt.

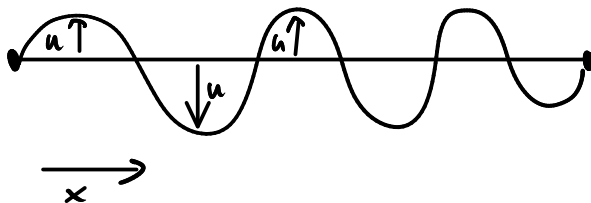
Many problems involve functions of several variables
 Then we are dealing with partial differential equations

Heat conduction: $T(x,t)$ temperature in rod



Heat Equation: $\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$

Wave propagation: $u(x,t)$ = displacement of vibrating string



Wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

Fluid flow

\vec{v} = velocity vector field

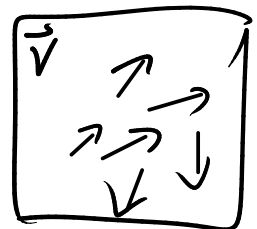
ϕ = pressure

k = viscosity \vec{f} = external force

(1) $\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = k \nabla^2 \vec{v} - \nabla \phi + \vec{f}$

(2) $\nabla \cdot \vec{v} = 0$

(3) $\vec{v}(x, 0) = \vec{v}_0(x)$ initial condition



Navier-Stokes

\$1000000 prize

see claymath.org