Example Find $\int_1^e \frac{\ln x}{x}\,dx$

Here it is not easy to determine what the inner function is. Your thinking process should go asking questions such as: Do we know the anti derivative of $\ln x$ or any function whose derivative is $\ln x$? Since the answer is No to these questions try using $\ln x$ as your u. Let $u = \ln x$ then $du = \frac{1}{x}dx$. Also for $x = 1 \Rightarrow u = \ln 1 = 0$ and $x = e \Rightarrow u = \ln e = 1$

$$\begin{aligned}\int_1^e \frac{\ln x}{x}\,dx &= \int_1^e \ln x \cdot \frac{1}{x}\,dx \\ &= \int_0^1 u\,du = \int_0^1 u\,du \\ &= \frac{u^2}{2}\Big]_0^1 = \frac{1}{2}\end{aligned}$$

Graphically you can see below that the area is transformed by our substitution into a triangular area on the second figure much easier to calculate even without calculating an integral.