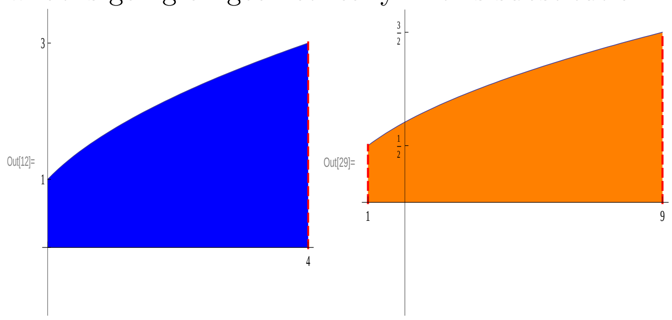
Before evaluating the integral let's take a little stop here and look closer what is going on geometrically in this substitution



The substitution transformed area in the first figure to the area in the second one. The equality of the above two integrals says the area has not changed but the substitution only turned our integral into an integral easier to evaluate using Fundamental Theorem of Calculus.Now let's go and calculate the integral

$$\begin{aligned}\int_0^4 \sqrt{2x+1}\,dx &= \frac{1}{2}\int_1^9 \sqrt{u}\,du \\ &=_{FTC} \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^9 \\ &= \frac{1}{2}\left[\frac{2}{3}(9)^{3/2} - \frac{2}{3}(1)^{3/2}\right] \\ &= \frac{1}{2}\left[18 - \frac{2}{3}\right] = \frac{26}{3}\end{aligned}$$

<u>Remark</u> In a definite integral, the "dx" is much more than a place holder. It says, "integrate in the positive x-direction".