**The Substitution Rule** If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f'(g(x)) \cdot g'(x)\,dx = \int f(u)\,du$$

<u>Caution</u> Note that after the substitution you have NO x's left inside the integral. The new variable on the right hand side is "u".

**Example** Find $\int x^3 \cos(x^4 + 2)\,dx$

Here the "inner" function is $x^4 + 2$ that will be our u, i.e. $u = x^4 + 2$. Then find the differential of u: $du = 4x^3\,dx$. Since you only have $x^3\,dx$ in your integral, divide both sides of the differential by 4 to get what you exactly need to replace $x^3\,dx$ with.

$$\begin{aligned}
\int x^3 \cos(x^4 + 2)\,dx &= \int \cos(x^4 + 2)x^3\,dx \\
&= \int \cos u \cdot \frac{1}{4}\,du \\
&= \frac{1}{4}\int \cos u\,du \\
&= \frac{1}{4}\sin u + C(\text{ go back to your original variable }) \\
&= \frac{1}{4}\sin(x^4 + 2) + C
\end{aligned}$$

**Example** Calculate $\int e^{5x}\,dx$

The "inner" function here is $5x$ it is inside the exponential function after all. So $u = 5x$, differential $du = 5\,dx$. Divide both sides by 5 since you only need $dx$ so $\frac{1}{5}du = dx$.

$$\begin{aligned}
\int e^{5x}\,dx &= \int e^u(\frac{1}{5}\,du) \\
&= \frac{1}{5}\int e^u\,du \\
&= \frac{1}{5}e^u + C(\text{ go back to your original variable }) \\
&= \frac{1}{5}e^{5x} + C
\end{aligned}$$