Computing the Integral

Let f be continuous on $[a, b]$ and let $\Phi(x) = \int_a^x f(s)\,ds$ for $a \leq x \leq b$. Now let F be any function st $F'(x) = f(x)$ for $a \leq x \leq b$, (i.e. F is an anti-derivative of f.) Then since F and Φ have the same derivative on $[a, b]$
$\Phi(x) - F(x) = c$
for some constant c. Since $\Phi(a) = 0$, then $0 - F(a) = c \Rightarrow c = -F(a)$. Hence $\Phi(x) - F(x) = -F(a)$ or $\Phi(x) = F(x) - F(a)$ and more importantly,
$\Phi(b) = \int_a^b F(b) - F(a)$.
This observation gives us the 2nd version of Fundamental Theorem of calculus

Theorem (2nd Version of Fundamental Theorem of Calculus) Let f be continuous on $[a, b]$ and let F be any anti-derivative of f on this interval then

$$\boxed{\int_a^b f(x)\,dx = \int_a^b F'(x)\,dx = F(b) - F(a)}$$

Remark The right hand side of the FTC-2 above could be also written as $F(x)|_a^b$

Example Compute $\int_0^1 (3x - 4x^3)\,dx$ and interpret is a signed area.

$F(x) = \frac{3}{2}x^2 - x^4$ is an anti-derivative of $f(x) = 3x - 4x^3$. Then by the FTC-2

$$\int_0^1 (3x - 4x^3)\,dx = \frac{3}{2}x^2 - x^4|_0^1 = (\frac{3}{2} - 1) - (0 - 0) = \frac{1}{2} = A_1 - A_2$$

where A_1 and A_2 represents the areas in the figure below