Variations on $\frac{d}{dx}\int_a^x f(s)\,ds = f(x)$

1) $\frac{d}{dx}\int_x^a f(s)\,ds = -\frac{d}{dx}\int_a^x f(s)\,ds = -f(x)$

2)

$$\begin{aligned}
\frac{d}{dx}\int_{u(x)}^{v(x)} f(s)\,ds &= \frac{d}{dx}\left(\int_c^{v(x)} f(s)\,ds + \int_{u(x)}^{c} f(s)\,ds\right)\\
&= \frac{d}{dx}\left(\underbrace{\int_c^{v(x)} f(s)\,ds}_{\Phi(v(x))} - \underbrace{\int_c^{u(x)} f(s)\,ds}_{\Phi(u(x))}\right)\\
&= \frac{d}{dx}\left(\Phi(v(x)) - \Phi(u(x))\right)\\
&= \frac{d}{dx}\Phi(v(x)) - \frac{d}{dx}\Phi(u(x))\\
&= \Phi'(v(x))\cdot v'(x) - \Phi'(u(x))\cdot u'(x) \text{ by Chain Rule}\\
&= f(v(x))\cdot v'(x) - f(u(x))\cdot u'(x)
\end{aligned}$$

Example Let $q(x) = \int_0^{\sin x}(1-s^2)\,ds$. Find $q'(x)$.

$q'(x) = \frac{d}{dx}\int_0^{\sin x}(1-s^2)\,ds = (1-\sin^2 x)\cdot(\cos x) = \cos^3 x$ by the Variation 2 above.

Example Let $g(x) = \int_x^{x^3}\frac{1}{s}\,ds$. Find $g'(x)$.
$g'(x) = \frac{d}{dx}\int_x^{x^3}\frac{1}{s}\,ds = \frac{1}{x^3}\cdot(3x^2) - \frac{1}{x}\cdot 1 = \frac{3}{x} = \frac{1}{x} = \frac{2}{x}$
Example Let $F(x) = \int_0^x \frac{t-1}{1+t^2}\,dt$. Find the critical numbers of F and at each critical number determine if F has a local max, min or neither.
By FTC-1 we know $F'(x) = \frac{x-1}{1+x^2}$. $F'(x) = 0 \Rightarrow x = 1$. $F'(x)$ dne is not possible. Since $F''(1) = \frac{-1^2+2+1}{(1+1^2)^2} = \frac{1}{2} > 0$ at $x = 1$, F has a local min.