9) Use Properties of integrals to verify the inequality below without evaluating the integral.

$$\int_0^{\frac{\pi}{2}} x \sin x \, dx \leq \frac{\pi^2}{8}$$

We will use Property 6 to prove the result. Since $\sin x \leq 1$ for x in $[0, \frac{\pi}{2}]$. So $x \sin x \leq x \cdot 1 = x$ on this interval. Hence
$\int_0^{\frac{\pi}{2}} x \sin x \, dx \leq \int_0^{\frac{\pi}{2}} x \, dx \leq= \frac{(\frac{\pi}{2})^2}{2} - \frac{0^2}{2} = \frac{\pi^2}{8}$

10) Use Property 7 to evaluate the integral $\int_{-3}^{0} (x^2 + 2x) \, dx$
To use Property 7 again we need to find the lower and upper bounds m and M for the function $y = x^2 + 2x$. As before we could use the graph of this function to figure these bounds but instead I would like to use Calculus and what we learnt so far to show another way of finding these bounds. Finding m and M can be thought of finding the absolute maximum and minimum of $y = x^2 + 2x$ over the interval $[-3, 0]$. Since we have a continuous polynomial function we may use EVT and we know $y = x^2 + 2x$ has an absolute maximum and minimum on this interval. So $f'(x) = 2x + 2 = 0 \Rightarrow x = -1$ then comparison gives $f(-3) = 9 - 6 = 3$ absolute max
$f(-1) = 1 - 2$ absolute min
$f(0) = 0$
Hence $-1 \leq x^2 + 2x \leq 3$ on [-3,0]. Using Property 7 now we get

$$-1(0+3) \leq \int_{-3}^{0} (x^2 + 2x) \, dx \leq 3(0+3) \Rightarrow -3 \leq \int_{-3}^{0} (x^2 + 2x) \, dx \leq 9$$