7) Use Properties of integrals to verify the inequality below without evaluating the integrals.

$$\int_1^2 \sqrt{5-x}\, dx \geq \int_1^2 \sqrt{x+1}\, dx$$

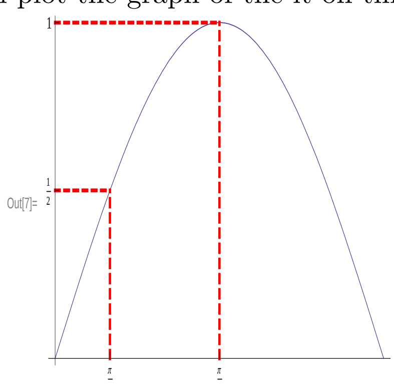
We will use the Property 6 above to prove the inequality. So we have to show $\sqrt{5-x} \geq \sqrt{x+1}$ on the interval $[1,2]$.And we show this below: On this interval

$$2 \geq x \Rightarrow 2{+}2 \geq x{+}x \Rightarrow 5{-}1 = 4 \geq x{+}x \Rightarrow 5{-}x \geq x{+}1 \Rightarrow \sqrt{5-x} \geq \sqrt{x+1}$$

Note that we may take the square root of both sides because
$x \geq 1 \Rightarrow x+1 \geq 2 > 0$
Hence by Property 6 $\int_1^2 \sqrt{5-x}\, dx \geq \int_1^2 \sqrt{x+1}\, dx$.
8) Use Properties of integrals to verify the inequality below without evaluating the integrals.

$$\frac{\pi}{6} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x\, dx \leq \frac{\pi}{3}$$

We will use the Property 6 to verify this inequality. And we need to find the two bounds m and M first to be able to use this property. Since the integral we are trying to find bounds for is for $\sin x$ on the interval $[\frac{\pi}{6}, \frac{\pi}{2}]$ we will plot the graph of the it on this interval.



From the graph we observe that $\frac{1}{2} \leq \sin x \leq 1$ on $[\frac{\pi}{6}, \frac{\pi}{2}]$. So $m = \frac{1}{2}$ and $M = 1$. Then by Property 6
$\frac{1}{2}(\frac{\pi}{2} - \frac{\pi}{6}) \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x\, dx \leq 1 \cdot (\frac{\pi}{2} - \frac{\pi}{6}) \Rightarrow \frac{\pi}{6} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x\, dx \leq \frac{\pi}{3}$