Properties of $\int_a^b f(x)\,dx$

Suppose $\int_a^b f(x)\,dx$ and $\int_a^b g(x)\,dx$ exists then

1) $\int_a^b cf(x)\,dx$ exists and $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
2) $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
3) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
4) $\int_a^a f(x)\,dx = 0$
5) If $f(x) \geq 0$ on $[a,b]$ then $\int_a^b f(x)\,dx \geq 0$
6) If $f(x) \geq g(x)$ on $[a,b]$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$
7) If $m \leq f(x) \leq M$ on $[a,b]$ then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$
8) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ where $a < c < b$

We will evaluate couple of integrals next using the properties of definite integrals.

Examples 1)$\int_0^1 (x-5)\,dx = \int_0^1 x\,dx - \int_0^1 5\,dx = \frac{1^2-0^2}{2} - 5(1-0) = \frac{-9}{2}$ by Property 2
2) $\int_0^2 6x^2 = 6\int_0^2 x^2\,dx = 6\frac{2^3-0^3}{3} = 16$ by Property 1
3) $\int_2^0 5\,dx = -\int_0^2 5\,dx = -5(2-0) = -10$ by Property 3
4) $\int_2^2 x^3 \sin^{-1} x\,dx = 0$ by Property 4
5) Write the given difference of integrals as a single integral of the form $\int_a^b f(x)\,dx$.

$$\int_2^{10} f(x)\,dx - \int_2^7 f(x)\,dx$$

Use Property 8 to write the first integral as $\int_2^{10} f(x)\,dx = \int_2^7 f(x)\,dx + \int_7^{10} f(x)\,dx$ then
$\int_2^{10} f(x)\,dx - \int_2^7 f(x)\,dx = \int_2^7 f(x)\,dx + \int_7^{10} f(x)\,dx - \int_2^7 f(x)\,dx = \int_7^{10} f(x)\,dx$
6) If $\int_0^1 f(t)\,dt = 2$, $\int_0^4 f(t)\,dt = -6$ and $\int_3^4 f(x)\,dx = 1$, find $\int_1^3 f(t)\,dt$.
First note that $\int_1^3 f(t)\,dt = \int_0^3 f(t)\,dt - \int_0^1 f(t)\,dt$
And since $\int_0^4 f(t)\,dt = \int_0^3 f(t)\,dt + \int_3^4 f(t)\,dt$ by Property 8
$\int_0^3 f(t)\,dt = -6 - 1 = -7$, $\int_1^3 f(t)\,dt = -7 - 2 = -9$