Example Show that $\boxed{\int_a^b k\,dx = k(b-a)}$ for any constant k.

We will use the same Δx as before i.e $\Delta x = \frac{b-a}{n}$ and the right hand endpoint of each subinterval as $c_i^\star = c_i$

$$\sum_{i=1}^{n} f(c_i^\star)\Delta x = \sum_{i=1}^{n} k \cdot \frac{b-a}{n}$$
$$= k\frac{b-a}{n} \cdot n = k(b-a) \to k(b-a) \text{ as } n \to \infty$$

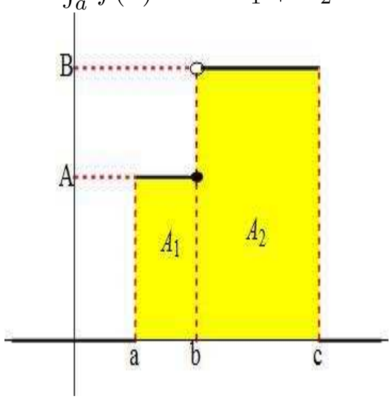
Example Show that $\boxed{\int_a^b x^2\,dx = \frac{b^3-a^3}{3}}$

This is left as an exercise for you to prove.

Cautionary Example Let $f(x) = \begin{cases} A & \text{if } a \le x \le c \\ B & \text{if } c < x \le b \end{cases}$ where $A < B$.

Does $\int_a^b f(x)\,dx$ exists?

Note that f is discontinuous at $x = c$. So our theorem does not promise us any result. But looking at the graph of f below we see that $\int_a^b f(x)\,dx$ exits and $\int_a^b f(x)\,dx = A_1 + A_2 = A(c-a) + B(b-c)$



Next we will talk about a little what does "negative area" mean?

If f takes both positive and negative values as in the figure below, then the Riemann Sum is the sum of the area of the rectangles that lie above the x-axis, and the *negatives* of the areas of the rectangles that lie below the x-axis.