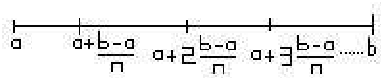

Choose $c_i^\star = c_{i-1}$ Left-hand endpoint of each subinterval.

$$\begin{aligned}
\sum_{i=1}^{n} f(c_i^\star)\Delta x &= \sum_{i=1}^{n} c_{i-1}\cdot\frac{b-a}{n}\\
&= \sum_{i=1}^{n}\left(a+(i-1)\frac{b-a}{n}\right)\cdot\frac{b-a}{n}\\
&= \sum_{i=1}^{n} a\cdot\frac{b-a}{n} + \sum_{i=1}^{n}\left(\frac{b-a}{n}\right)^2(i-1)\\
&= \frac{a(b-a)}{n}n + \sum_{i=1}^{n}\left(\frac{b-a}{n}\right)^2 i - \sum_{i=1}^{n}\left(\frac{b-a}{n}\right)^2\\
&= a(b-a) + \left(\frac{b-a}{n}\right)^2\frac{n(n+1)}{2} - n\left(\frac{b-a}{n}\right)^2\\
&= a(b-a) + \frac{(b-a)^2}{2}\frac{n+1}{n} - \frac{(b-a)^2}{n}
\end{aligned}$$

Take the limit $n\to\infty$

$$\begin{aligned}
\int_a^b x\,dx &= \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i^\star)\Delta x\\
&= \lim_{n\to\infty}\left(a(b-a) + \frac{(b-a)^2}{2}\frac{n+1}{n} - \frac{(b-a)^2}{n}\right)\\
&= a(b-a) + \frac{(b-a)^2}{2}\cdot 1 - 0\\
&= (b-a)\left[a+\frac{b-a}{2}\right] = (b-a)\left[\frac{2a+b-a}{2}\right]\\
&= (b-a)\frac{b+a}{2} = \frac{b^2-a^2}{2}
\end{aligned}$$