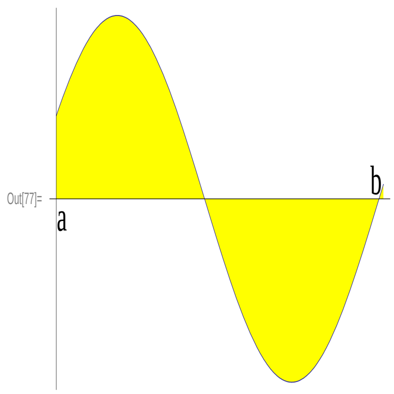
Now we will calculate the definite integrals using the definition and the tip in the second Remark, but we need a tool that will secure the existence of the integral first. And here it comes;
Theorem If f is continuous on $[a, b]$, then it is integrable on $[a, b]$.

For any curve that is continuous the area under it over a closed interval $[a, b]$ has to be finite and also term $f(i)\Delta x$ in the Riemann Sum will always make sense i.e. defined



Example Calculate $\int_a^b x\,dx$ using the definition of definite integral

By the theorem above we know this integral exists because $y = x$ is continuous everywhere hence on $[a, b]$. As we noted in Remark 2 after the definition of the definite integral since the limit exists if we choose a specific Δx and special $c_i^\star$'s the limit should be equal to $\int_a^b x\,dx$. Why? Because we can think of the Riemann Sum we got by our special choice of Δx and $c_i^\star$'s as a "subsequence" of the general Riemann sum. And since the general one converges, the subsequence has to converge to the same limit, i.e the value of the integral.
We will choose $\Delta x = \frac{b-a}{n}$ and in the below cases we will choose specific $c_i^\star$'s.Here is how our interval looks like