Since $\lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$ by the Squeeze Theorem

$$\lim_{n\to\infty} L_n = \frac{1}{3} \leq \lim_{n\to\infty} A \leq \lim_{n\to\infty} R_n = \frac{1}{3}$$

implies $A = \frac{1}{3}$.

A very good questions to raise at this point would be: In both of the calculations we have chosen the "height" of the rectangles to be specific points, meaning we have chosen them either the left end point or the right end point of the interval. Hence our summation calculations were easy. What will happen to the whole argument above if we let the choice of "height-producing points" in each subinterval to be completely free? Well this is exactly ho we will define the area under the curve.

Definition Let n be a positive integer and let $\{x_0, x_1, x_2, ..., x_n\}$ be a set of points equally spaced in the interval $[a, b]$. Define $\Delta x = x_i - x_{i-1} = \frac{b-a}{n}$ and choose any points $c_1, c_2, c_3, ...c_n$ where c_i is any number in the subinterval $[x_i, x_{i-1}]$. Then the **Riemann Sum** that approximates the area under the curve of $y = f(x)$ on the interval $[a, b]$ is

$$\sum_{i=1}^{n} f(c_i)\Delta x$$