

Example Calculate $\sum_{i=1}^7 (2i + 3) - \sum_{k=2}^6 (k + 4)$

$$\begin{aligned}\sum_{i=1}^7 (2i + 3) - \sum_{k=2}^6 (k + 4) &= \sum_{i=1}^7 (2i + 3) - \sum_{i=1}^5 (i + 5) \text{ Let } i=k-1 \text{ in the 2nd sum} \\ &= \underbrace{\sum_{i=1}^5 (2i + 3)}_{= \sum_{i=1}^5 2i + 3 - i - 5} - \underbrace{\sum_{i=1}^5 (i + 5)}_{= \sum_{i=1}^5 i - 2} + \sum_{i=6}^7 (2i + 3) \\ &= \sum_{i=1}^5 2i + 3 - i - 5 + \sum_{i=6}^7 (2i + 3) \\ &= \sum_{i=1}^5 i - \sum_{i=1}^5 2 + \sum_{i=6}^7 (2i + 3) \\ &= \frac{5 \cdot 6}{2} - 2 \cdot 5 + (2 \cdot 6 + 3) + (2 \cdot 7 + 3) = 37\end{aligned}$$