Theorem Some basic sums

1) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ Sum of the first n positive integers

2) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ Sum of the first n^2 integers

3) $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Example Calculate $\sum_{i=1}^{5}(2i^2 - 4i + 11)$

$$\begin{aligned}
\sum_{i=1}^{5}(2i^2 - 4i + 11) &= 2\sum_{i=1}^{5} i^2 - 4\sum_{i=1}^{5} i + \sum_{i=1}^{5} 11 \\
&= 2\left(\frac{5(5+1)(2\cdot 5+1)}{6}\right) - 4\left(\frac{5(5+1)}{2}\right) + 11\cdot 5 \\
&= 2(55) - 4(15) + 55 = 105
\end{aligned}$$

Example Calculate $\sum_{k=5}^{17}(2k^2 + 3k + 4)$

This problem is designed to show you how you can "shift" a sum by adjusting the index of summation and in turn adjusting the terms you are summing.

$$\begin{aligned}
\sum_{k=5}^{17}(2k^2 + 3k + 4) &= \sum_{j=1}^{13}(2(j+4)^2 + 3(j+4) + 4) \text{ We let j=k-4 here} \\
&= \sum_{j=1}^{13} 2j^2 + 19j + 48 \\
&= 2\sum_{j=1}^{13} j^2 + 19\sum_{j=1}^{13} j + \sum_{j=1}^{13} 48 \\
&= 2\frac{13\cdot 14\cdot 27}{2} + 19\frac{13\cdot 14}{2} + 13\cdot 48 = 3172
\end{aligned}$$